MOTION OF BODIES IN A GRANULAR BED

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The investigation of the motion of bodies in a granular bed with various physical properties is of considerable practical interest. In [1-3] and a number of other studies the motion of bodies in two-phase (solid phase – gas) media was considered. The method and results of calculating the penetration of rigid bodies traveling at high speeds into a soil mass, which in a particular case may be regarded as a granular medium, are described in [4]. The method of calculating the parameters of motion of the body given in [4] can be used when a shock wave is formed in the medium ahead of the moving body, which is characteristic of supersonic regimes. In [5] it was pointed out that the speed of sound in sand is of the order of 100 m/sec, and empirical relations were given for determining the deceleration of a projectile traveling at both subsonic and supersonic speeds. However, Allen et al. [5] give no data on the resistance of the sand, so that the known relations cannot be used for calculating the motion of a body in a granular medium under the action of an applied force. Among the known relations for calculating the resistance of sand it is worth noting that of Poncelet. The values of its empirical coefficients for the supersonic regime are given in [6].

Experimental data on the resistance of a granular medium to a body traveling through it are given, together with empirical formulas for the dependence of the resistance of the medium on its mechanical characteristics and the velocity of the body in the subsonic regime and the results of calculating the projection of bodies into a granular bed by a two-phase flow.

For investigating the effect of the velocity of the body on the resistance of the granular medium we used the experimental apparatus shown schematically in Fig. 1. A pipe consisting of two sections was horizontally connected to a box measuring 0.42 × 0.80 × 1.10 m filled with sand 1 with a bulk density of 1500 kg/m^3 and a characteristic particle diameter of 0.25 mm. Between the sections we installed a diaphragm 4. Into the section of the pipe connected to the box we introduced a wooden rod 6 with a piston and dynamometer. The other section was filled with sand 5. When the pore space of the sand in the pipe section was brought up to the predetermined pressure the diaphragm burst, as a result of which the rod was propelled into the bed of sand by the two-phase flow. The dynamometer was the 50 mm diameter moving head 7 which by way of a steel ball rested on the diaphragm of a DD-10 transducer connected across an ID-2I indicator to an N145 lightbeam oscillograph.

The velocity of the body in the bed was recorded by the oscillograph connected to the ID-2I indicator, which had a Tee-type inductance bridge inserted into its input circuit. Into one arm of the bridge we introduced a measuring coil with an inductance of 350 μH, and into the other a balancing coil wound on an SB-20 type core. Initially, the bridge was balanced. Ferrites 3 were located at intervals of 20 mm in the body of the wooden rod. As the rod was displaced, the ferrites successively entered the response field of the measuring coil, thereby unbalancing the bridge. The mismatch signal, converted by the ID-2I indicator, was recorded by the oscillograph. Thus, on the oscillogram we simultaneously recorded the resistance and the displacement of the rod. The velocity was calculated from the time intervals between the passage of the ferrites through the measuring coil and the known distance between them.

The resistance to the body traveling through the granular medium is the sum of the static and dynamic components. The static component can be determined from the specific (per unit cross-sectional area) resistance to the penetration of a stamp. In order to control the variation of the static component, in loading the sand into the box we tamped it in layers by applying specified loads.

The results of experimentally investigating the dependence of the resistance of the sand on the velocity of the body for various specific stamp penetration resistances are presented

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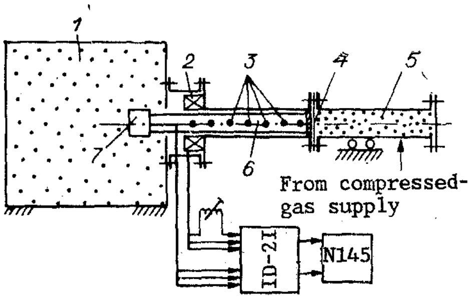

Fig. 1

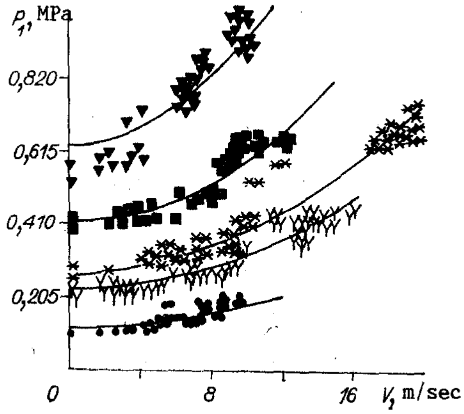

Fig. 2

in Fig. 2, where the experimental values of the resistance corresponding to specified values of the static component are indicated by identical symbols. A least squares analysis gave the following relation for calculating the resistance on the velocity interval from 0 to 20 m/sec:

$$P_1 = P_0(1 + 0{,}0044V^2), \tag{1}$$

where P_1 (MPa) is the specific (per unit cross-sectional area) resistance to the moving body, P_0 (MPa) is the static component of the specific resistance, and V (m/sec) is the velocity of the body.

The form of relation (1) corresponds to the well-known Poncelet formula, differing from it with respect to the numerical value of the velocity coefficient. The values of P_1 calculated from (1) are given in Fig. 2.

In order to calculate the parameters of the motion of a body propelled horizontally into a bed of sand by a two-phase medium we used the well-known equilibrium flow model [7]. The presence of a moving piston makes it necessary to calculate in moving coordinates. In this case instead of the coordinates t, x we introduce the coordinates τ, η determined from the formulas $\tau = t$, $\eta = x/\psi(t)$ [$\psi(t)$ is the coordinate of the piston], and the partial derivatives are related by the expressions

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} - \frac{\eta\dot{\psi}}{\psi}\frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial x} = \frac{1}{\psi}\frac{\partial}{\partial \eta}.$$

Then the system of differential equations for the equilibrium flow of a two-phase medium reduces to the form:

$$\begin{gathered} \frac{\partial R}{\partial \tau} = -\frac{\partial RU}{\partial \eta}, \quad \frac{\partial Ru}{\partial \tau} = -\frac{\partial (RuU + P)}{\partial \eta}, \quad \frac{\partial Re}{\partial \tau} = -\frac{\partial (ReU + Pu)}{\partial \eta}, \\ P = (k-1)\left(Re - \frac{Ru^2}{2}\right)\frac{1}{\psi}\,\frac{1}{1 + \dfrac{C_2}{C_1}\dfrac{1-\Pi_0}{\Pi_0}\dfrac{\rho_2}{(\rho_1)_0}}\,\frac{1}{1-(1-\Pi_0)\dfrac{\rho}{\rho_0}}, \\ R = \rho\psi, \; U = (u - \eta\dot{\psi})/\psi. \end{gathered} \tag{2}$$

Here, ρ and u are the density and velocity of the medium, e is the total energy per unit mass of the medium, P and k are the pressure and specific heat ratio of the gas, Π_0 is the initial porosity of the sand in the pipe, C_2 and C_1 are the specific heats of the sand and the gas, ρ_2 is the density of the solid phase, and $(\rho_1)_0$ and ρ_0 are the initial densities of the gas and the two-phase medium.

In order to determine the velocity and the coordinate of the piston, and hence the velocity $\dot{\psi}$ and the coordinate ψ of the calculation point moving with it, we used the equation of motion of the piston

$$d\dot{\psi}/d\tau = \Delta P f/m, \; d\psi/d\tau = \dot{\psi}, \tag{3}$$

where ΔP is the pressure difference across the piston, equal to the difference between the gas pressure behind the piston and the specific resistance of the sand determined from (1), and f and m are the area and mass of the piston together with the rod and the dynamometer. The system of equations (2) was integrated by a finite-difference method using (3).

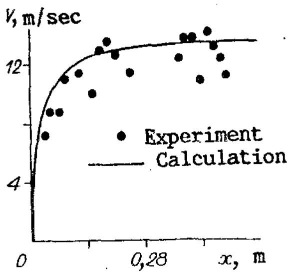

Fig. 3

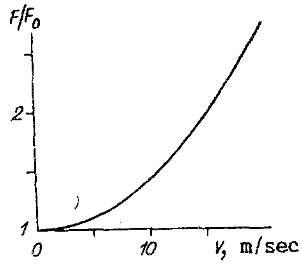

Fig. 4

In Fig. 3 we have plotted the experimental and calculated variation of the velocity of a body propelled by a two-phase medium into a bed of sand for the following parameters: length of sand-filled pipe 1.4 m, diameter 40 mm, initial porosity of sand in pipe 0.3, initial pore gas pressure 3.5 MPa, mass of rod with piston and dynamometer 1.64 kg, diameter of dynamometer 50 mm, static component of the resistance of the sand 0.62 MPa. Figure 4 shows the dependence of the relative (to the static component) resistance of the sand F/F_0 on the velocity of the body V.

On the basis of the result obtained we may conclude that at penetration velocities up to 3 m/sec the dynamic component of the resistance does not exceed 5% of the static and, without serious error, the problem of the penetration of a body into sand can be solved without taking the dynamic component into account. However, at velocities of 10-20 m/sec the resistance of the sand may exceed the value of the static component by a factor of 1.4-2.6.

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